[5] Theory of equation

[5.3] Relation Between Roots and Coefficients

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[5.3.1] Relations between the roots and coefficients

Let $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ be the roots of the polynomial equation $a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_{n-1} x + a_n = 0$ — (1)

Then,

$$\sum \alpha_1 = \alpha_1 + \alpha_2 + \alpha_3 + \dots + \alpha_n = -\frac{a_1}{a_0}$$

$$\sum \alpha_1 \cdot \alpha_2 = \alpha_1 \cdot \alpha_2 + \alpha_1 \cdot \alpha_3 + \alpha_2 \cdot \alpha_3 + \dots + \alpha_{n-1} \cdot \alpha_n = \frac{a_2}{a_0}$$

$$\sum \alpha_1 \cdot \alpha_2 \cdot \alpha_3 = \alpha_1 \cdot \alpha_2 \cdot \alpha_3 + \alpha_1 \cdot \alpha_2 \cdot \alpha_4 + \dots + \alpha_{n-2} \cdot \alpha_{n-1} \cdot \alpha_n = -\frac{a_3}{a_0}$$

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$$\alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdots \alpha_n = (-1)^n \cdot \frac{a_n}{a_0}$$

Working rule:-

(i) Sum of roots $= -\dfrac{\text{coefficient of } x^{n-1}}{\text{Coefficient of } x^n}$

(ii) Sum of products of roots taken two by two

$$= \frac{\text{coefficient of } x^{n-2}}{\text{Coefficient of } x^n}$$

(iii) Sum of products of roots taken three by three

$$= -\frac{\text{coefficient of } x^{n-3}}{\text{Coefficient of } x^n}$$

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(iv) Product of all the roots $= (-1)^n \cdot \dfrac{\text{Constant}}{\text{Coefficient of } x^n}$.

(a) For quadratic ~~polyn~~ equation, $f(x) = ax^2 + bx + c = 0$ has two roots α and β

$$\alpha + \beta = \frac{-b}{a}, \quad \alpha\beta = \frac{c}{a}.$$

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(b) For cubic equation $ax^3+bx^2+cx+d=0$ has three roots α, β, γ.

$$\alpha+\beta+\gamma = \frac{-b}{a}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

(c) For biquadratic equation

$ax^4+bx^3+cx^2+dx+e=0$ has four roots $\alpha, \beta, \gamma, \delta$

$$\alpha+\beta+\gamma+\delta = -\frac{b}{a}$$

$$\alpha\beta+\beta\gamma+\gamma\delta+\alpha\delta = \frac{c}{a}$$

$$\alpha\beta\gamma+\alpha\beta\delta+\beta\gamma\delta+\alpha\gamma\delta = -\frac{d}{a}$$

$$\alpha\beta\gamma\delta = -\frac{e}{a}$$

Example (1) :- Solve the equation $x^3-9x^2+14x+24=0$, two of whose roots are in the ratio of $2:3$.

Solution :-

$$f(x) = x^3-9x^2+14x+24 = 0 \quad \text{—(i)}$$

Let the roots of the equation be $2\alpha, 3\alpha, \beta$.

We have,

$$2\alpha+3\alpha+\beta = -\frac{(-9)}{1} = 9 \Rightarrow 5\alpha+\beta = 9 \quad \text{—(ii)}$$

$$2\alpha\cdot 3\alpha + 2\alpha\cdot\beta + 3\alpha\cdot\beta = \frac{14}{1} = 14$$

$$\Rightarrow \cancel{6\alpha^2+2\alpha\beta+3\alpha\beta=14}$$

$$\Rightarrow 6\alpha^2+5\alpha\beta = 14 \quad \text{—(iii)}$$

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And, $2\alpha \cdot 3\alpha \cdot \beta = \frac{-24}{1}$

$\cancel{6}\alpha^2\beta = -\cancel{24}^{4} \Rightarrow \alpha^2\beta = -4$ —— (iv)

Multiplying (ii) by 5α and subtract it from (iii)

$$5\alpha \cdot (5\alpha + \beta) - (6\alpha^2 + 5\alpha\beta) = 45\beta - 14$$

$$\Rightarrow 25\alpha^2 + \cancel{5\alpha\beta} - 6\alpha^2 - \cancel{5\alpha\beta} = 45\beta - 14$$

$$\Rightarrow 19\alpha^2 - 45\beta + 14 = 0$$

$$\therefore \alpha = \frac{45 \pm \sqrt{45^2 - 4 \cdot 19 \cdot 14}}{38}$$

$$= \frac{45 \pm 31}{38} = \frac{76}{36} \text{ or } \frac{14}{36} = 2 \text{ or } \frac{7}{19}$$

If $\alpha = 2$ then from (ii) $5\alpha + \beta = 9$

$5 \times 2 + \beta = 9 \Rightarrow \beta = 9 - 10 = -1$

If $\alpha = \frac{7}{19}$ then from (ii) $5\alpha + \beta = 9$

$5 \times \frac{7}{19} + \beta = 9 \Rightarrow \beta = 9 - \frac{35}{19} = \frac{171-35}{19}$

$= \frac{136}{19}$

But $\alpha = \frac{7}{19}$ and $\beta = \frac{136}{19}$ do not satisfy (iv)

Hence, we take $\alpha = 2$ then

roots are 4, 6 and -1

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